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# Critical regime of two-dimensional Ando model: relation between critical conductance and fractal dimension of electronic eigenstates

P Markoš<sup>1</sup> and L Schweitzer<sup>2</sup>

<sup>1</sup> Institute of Physics, Slovak Academy of Sciences, Dúbravská cesta 9, 854 11 Bratislava, Slovakia

<sup>2</sup> Physikalisch-Technische Bundesanstalt, Bundesallee 100, 38116 Braunschweig, Germany

E-mail: [peter.markos@savba.sk](mailto:peter.markos@savba.sk) and [ludwig.schweitzer@ptb.de](mailto:ludwig.schweitzer@ptb.de)

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## Abstract

The critical two-terminal conductance  $g_c$  and the spatial fluctuations of critical eigenstates are investigated for a disordered two-dimensional model of non-interacting electrons subject to spin-orbit scattering (Ando model). For square samples, we verify numerically the relation  $\sigma_c = 1/[2\pi(2 - D(1))]e^2/h$  between critical conductivity  $\sigma_c = g_c = (1.42 \pm 0.005)e^2/h$  and the fractal information dimension of the electron wave function,  $D(1) = 1.889 \pm 0.001$ . Through a detailed numerical scaling analysis of the two-terminal conductance we also estimate the critical exponent  $\nu = 2.80 \pm 0.04$  that governs the quantum phase transition.

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(Some figures in this article are in colour only in the electronic version)

## 1. Introduction

Quantum phase transitions in two-dimensional (2D) disordered systems have been the subject of continuing research for many decades. In particular, systems of non-interacting particles that are invariant under time reversal, but lack the symmetry of spin rotation are of special importance due to their property of a complete Anderson transition [1–6]. Thus, 2D systems with spin-orbit interaction, which belong to the symplectic symmetry class [7], exhibit an energy spectrum with regions of either localized or extended states separated by quantum critical points at which the correlation length diverges according to  $\xi(E) \sim |E - E_c|^{-\nu}$  with a universal exponent  $\nu$ . The energetical position of these points  $E_c$  depend on disorder strength and spin-orbit interaction. The corresponding critical eigenstates were found to be multi-fractal objects showing strong amplitude fluctuations as well as long-range spatial and energetical correlations [8–10].

Our knowledge about the critical properties like critical energy and disorder, or about the exponents that govern their scaling, originate mainly from numerical investigations of quantities like localization length [3, 4, 6, 12–14] or spectral correlations of eigenvalues [15] both of which are not directly accessible to experimental detection. The aim of the present paper is to present scaling results of the two-terminal conductance [16], a physical quantity that can easily be measured in experiment, and to examine a recently proposed relation between the average critical conductance and the generalized fractal dimensions of critical eigenstates. Therefore, we first present our numerical data for the two-terminal conductance and obtain the critical parameters of the model by a finite-size scaling analysis. In the metallic regime, the mean conductance is expected to increase with increasing system size while a decrease is usually found in the localized regime. At the critical point, the mean conductance  $g_c$  should be a universal, system size independent quantity and equal to the critical conductivity  $\sigma$ . Hence, the size scaling of our numerical data enables us to estimate the critical disorder  $W_c$ , and to calculate the critical conductance  $g_c$ . We also find the critical exponent  $\nu$  which governs the energy and disorder dependence of the conductance near the critical point.

Then we discuss the fractal properties of critical wave functions and calculate fractal dimensions  $D(q)$ . Non-uniform spatial amplitude distributions of electron wave functions that exhibit multi-fractal properties are a characteristic feature of the critical regime at metal–insulator transitions. It is well known that the corresponding generalized multi-fractal dimensions influence the time-dependent transport properties [11]. Here, we show that they also determine the conductance at a quantum critical point. Our main result is a confirmation of the relation  $D(q) = 2 - q/[\beta 4\pi^2 \varrho D\hbar]$  between the fractal dimensions and the conductivity (per spin direction),  $\sigma = e^2 \varrho D$ , proposed by Fal’ko and Efetov [17]. We also examine the validity of Janssen’s formula [18, 19] which relates the Lipschitz–Hölder exponent  $\alpha_0$  of the maximum of the multi-fractal distribution  $f(\alpha)$  to the scaling parameter  $\Lambda_c = 1/[\pi(\alpha_0 - d)]$  for quasi-1D systems. We find, however, our result for the symplectic Ando model ( $d = 2$ ) to be at variance with this proposal.

## 2. The model and methods

### 2.1. Hamiltonian

We study the two-dimensional (2D) Ando model [6] defined on a square lattice by the Hamiltonian

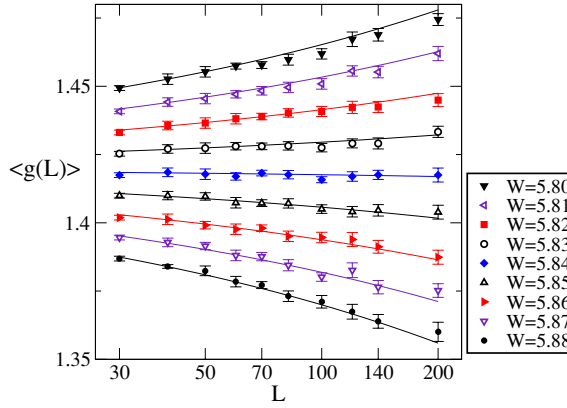
$$\mathcal{H} = W \sum_n \varepsilon_n c_n^\dagger c_n + \sum_{[nn']} V_{nn'} c_n^\dagger c_{n'}. \quad (1)$$

The  $\varepsilon_n$  are uncorrelated random on-site energies chosen according to a box probability distribution in the range  $|\varepsilon_n| \leq 1/2$ . The parameter  $W$  measures the strength of the disorder. The spin dependent hopping terms between nearest neighbour sites in the direction parallel and perpendicular to the current flow are

$$V_{\parallel} = V \begin{pmatrix} c & s \\ -s & c \end{pmatrix}, \quad \text{and} \quad V_{\perp} = V \begin{pmatrix} c & -is \\ -is & c \end{pmatrix}, \quad (2)$$

with  $c^2 + s^2 = 1$ . The parameter  $s$  determines hopping accompanied with a change of the spin of the electron. As usual, it is set to  $s = 0.5$ . The special case  $s = 0$  corresponds to a system with orthogonal symmetry which exhibits no metal–insulator transition in 2D. Energies are measured in units  $V = 1$  and lengths in units of the lattice constant  $a = 1$ .

It is well known that for Fermi energy  $E_F \approx 0$ , the Ando model exhibits a metal–insulator transition as a function of disorder at  $W = W_c \approx 5.8$  [6, 12]. The critical exponent for various



**Figure 1.** The mean conductance  $\langle g(L) \rangle$  as a function of the system size  $L$  for various values of disorder strength  $W$ . Within the interval of system sizes  $30 \leq L \leq 200$ ,  $\langle g(L) \rangle$  is almost constant for disorder  $W = 5.84$ . This indicates that finite-size effects are absent and irrelevant scaling terms are negligible. Solid lines are fits to (5) with critical parameters  $g_c = 1.42$ ,  $W_c = 5.838$ ,  $\nu = 2.80$  and  $A = -0.23$ .

2D systems with symplectic symmetry has been estimated in numerous works using different numerical methods [12, 14, 15, 20–23], leading to rather inconsistent results. The critical conductance was studied in [24]. We will discuss some of these results later together with our own results.

## 2.2. Conductance

The two-terminal conductance of a particular sample is calculated from

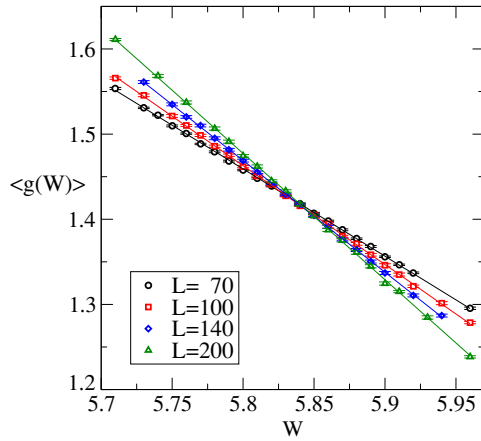
$$g = \text{Tr} t^\dagger t, \quad (3)$$

where  $t$  is the transmission matrix. Its elements  $t_{\alpha\beta}$  determine the transmission amplitude from channel  $\alpha$  into channel  $\beta$ . Two semi-infinite ideal leads are attached to the disordered sample and periodic boundary conditions are applied in the transverse direction. The Fermi energy is fixed at  $E_F = 0.01$  and spin-orbit scattering is assumed to be absent in the leads.

We use the algorithm of Pendry *et al* [25] for our numerical calculations. The size of the sample varied from  $L = 20$  to  $L = 200$ . We concentrate on the analysis of the  $L$ -dependence of the mean conductance in the neighbourhood of the critical point at  $W_c \sim 5.80$ . Because of the absence of self-averaging of the conductance, we need to analyse data for a large number of macroscopically identical samples which differ only by the respective microscopic realization of the disorder. For each disorder strength,  $5.71 \leq W \leq 5.96$ , and each system size, we collected a statistical ensemble of  $N_{\text{stat}} \geq 10^5$  samples and calculate the mean conductance  $\langle g \rangle$  and variance  $\text{var} g = \langle g^2 \rangle - \langle g \rangle^2$ . Here,  $\langle \dots \rangle$  means averaging over a statistical ensemble. Mean values  $\langle g \rangle$  with the uncertainty  $\delta g = (\text{var} g / N_{\text{stat}})^{1/2}$  are used in the scaling analysis. As a typical mean value  $\langle g \rangle \approx 1.4$  and  $\text{var} g \approx 0.36$ , the relative uncertainty  $\delta g / \langle g \rangle$  of our data is of order of 0.0015. Typical numerical data are shown in figures 1 and 2.

## 2.3. Critical wave function

Electron wave functions at quantum critical points of disordered systems exhibit a peculiar spatial structure [26, 27]. For our symplectic model, the critical wave functions for systems



**Figure 2.** Disorder dependence of the mean conductance  $\langle g(W) \rangle$  for  $L = 70, 100, 140$  and  $200$ . Solid lines are linear fits (6).

sizes of up to  $L = 260$  were obtained by direct diagonalization using a Lanczos algorithm. The fractality of the spatial fluctuations of the modulus of the normalized eigenstates  $\psi_E(r)$  was determined from the power-law scaling of the  $q$ th moment defined via a ‘box probability’

$$P(q, \lambda) = \sum_i^{N(l)} \left( \sum_{r \in \Omega_i(l)} |\psi_E(r)|^2 \right)^q \sim \lambda^{\tau(q)}. \quad (4)$$

The generalized fractal dimensions  $D(q) = \tau(q)/(q-1)$  or the so-called  $f(\alpha(q))$ -distribution was thus derived [28–30], where  $\Omega_i(l)$  is the  $i$ th box of size  $l = \lambda L$ . The  $\tau(q)$  and  $f(\alpha(q))$  are related by a Legendre transform.

#### 2.4. Scaling analysis

At the critical point  $W \equiv W_c$ , the conductance does not depend on the system size. In accordance with the scaling theory of localization [31], we assume that in the neighbourhood of the critical point the  $L$  dependence of  $g$  is governed by the critical exponent  $\nu$ ,

$$g_{\text{sc}}(W, L) = g_c + A(W - W_c)L^{1/\nu}. \quad (5)$$

We collected numerical data  $g(W, L)$  for more than  $N = 200$  values of  $W$  and  $L$  and obtain the critical parameters,  $g_c$ ,  $W_c$  and  $\nu$  from a fit of the numerical data to the scaling ansatz (5). We made sure that more sophisticated fits [32] which include higher order terms in disorder are not needed because our data exhibit already a perfect linear  $W$ -dependence (see figure 2). Also, subject to the uncertainty of our raw data, we found that possible irrelevant scaling fields [32, 33] are weak and do only marginally influence our results.

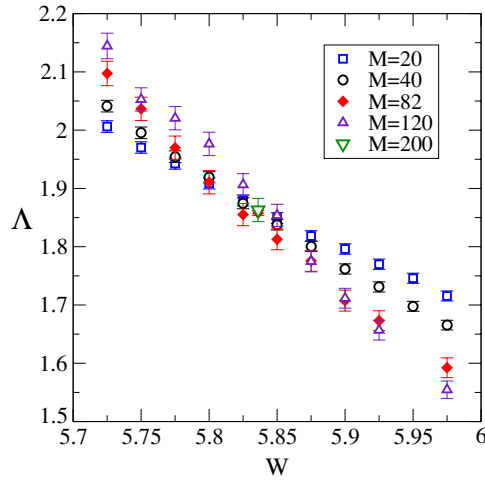
The most simple analysis of critical parameters can be done as follows: first, we calculate parameters of the linear  $W$ -dependence of the conductance

$$g(W, L) = g_0(L) + W g_1(L). \quad (6)$$

Comparing (6) with (5), we see that the slope  $g_1$  depends on the system size as

$$g_1(L) \propto L^{1/\nu}. \quad (7)$$

Then, a power-law fit  $g_1(L)$  versus  $L$  gives us the critical exponent.



**Figure 3.** The scaling parameter  $\Lambda$  calculated for quasi-1D geometry versus disorder strength for varying system width  $M$ . The data confirm a critical disorder  $W_c \approx 5.835$  and  $\Lambda_c \simeq 1.87 \pm 0.02$ .

We also minimize numerically the function

$$F = \frac{1}{N} \sum_{LW} \frac{1}{(\delta g)^2} [\langle g(W, L) \rangle - g_{sc}(W, L)]^2 \quad (8)$$

with respect to unknown parameters  $W_c$ ,  $\nu$  and  $g_c$  and  $A$ . To estimate the accuracy of the final result, we repeat  $N_m$  times the minimization of the function  $F$  with input data  $\langle g(W, L) \rangle$  randomly fluctuating within their respective error bars. To understand the role of the finite-size effects, we use in the scaling analysis only data for systems with sizes  $L_{\min} \leq L \leq L_{\max}$  and study how the obtained critical parameters depend on the choice of  $L_{\min}$  and  $L_{\max}$ .

### 3. Results and discussions

#### 3.1. Critical parameters

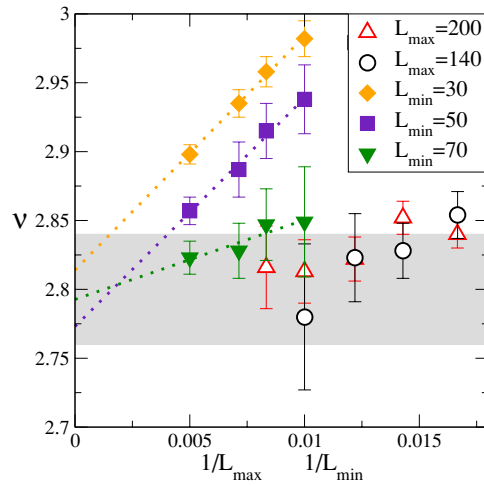
Our analysis showed that neither the critical disorder nor the critical conductance depend on the choice of the minimal and maximal system size,  $L_{\min}$  and  $L_{\max}$ . From all the data we concluded that

$$W_c = 5.838 \pm 0.007. \quad (9)$$

The estimate of the critical disorder differs considerably from the result of Fastenrath *et al* [12] ( $W_c = 5.74$ ), obtained from scaling analysis of the localization length of quasi-1D systems. Since previous results were derived from small system's widths, the discrepancy could be explained as an effect of finite-size corrections. To support this assumption, we performed numerical simulations for quasi-1D systems of size  $M \times L$ . Figure 3 shows the calculated data for the ratio  $\Lambda = \lambda_M/M$ , the localization length  $\lambda_M$  divided by the system width  $M$  [31, 34]. For our purposes, we run  $\Lambda$  with an accuracy of only 1%. This was sufficient to confirm that our estimation of the critical disorder derived from the conductance calculations indeed coincides with the one obtained from the localization length. Also, we get an estimate of the critical value  $\Lambda_c$  of the scaling parameter  $\Lambda$ ,

$$\Lambda_c \simeq 1.87 \pm 0.02 \quad (10)$$

which agrees well with a recent result of Asada *et al* [23] obtained for a  $SU(2)$  model.



**Figure 4.** The critical exponent  $\nu$  obtained from numerical data with  $L_{\min} \leq L \leq L_{\max}$ . Full symbols show how  $\nu$  depends on the choice of  $L_{\max}$  for fixed minimal system size  $L_{\min}$ . Open symbols show how  $\nu$  depends on  $L_{\min}$  for fixed  $L_{\max}$ ,  $L_{\max} = 140$  ( $\circ$ ), and  $L_{\max} = 200$  ( $\triangle$ ). The shaded area highlights our estimate of the critical exponent  $\nu = 2.80 \pm 0.04$ .

From our conductance data, the critical exponent  $\nu$  was obtained for the first time using finite-size scaling of the calculated electrical two-terminal conductance, which is, unlike the localization length, an easy way to measure physical quantity. With the finite-size scaling analysis described in section (2.4), our estimation of the critical exponent gives

$$\nu = 2.8 \pm 0.04, \quad (11)$$

which is close to a numerical result  $\nu = 2.746 \pm 0.009$  for a  $SU(2)$  model published recently [23], but considerably larger than  $\nu = 2.05 \pm 0.08$  [6] and smaller than  $\nu = 2.88 \pm 0.15$  as obtained in [21]. In contrast to the critical conductance and critical disorder, the critical exponent is sensitive to the system size as can be seen in figure 4. The estimate of  $\nu$  is therefore highly non-trivial and error bars obtained from various fit procedures differ.

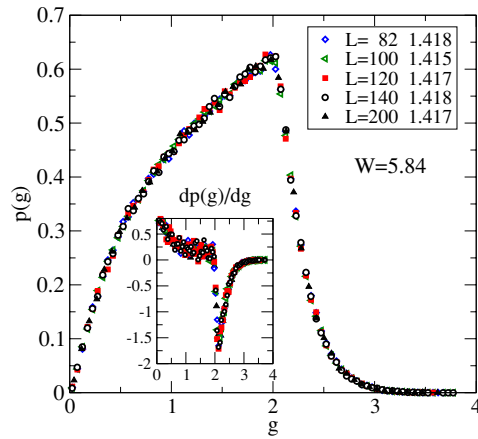
### 3.2. Critical conductance

The critical two-terminal conductance obtained by us for square samples with periodic boundary conditions in the transverse direction,

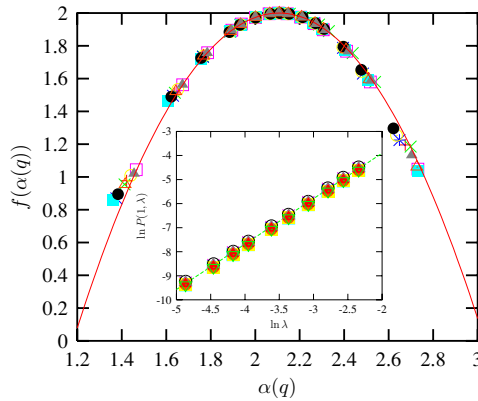
$$g_c = 1.42 \pm 0.005, \quad (12)$$

can be compared with the result for a  $SU(2)$  model studied recently in [24]. From the corresponding numerical data for size  $L \leq 97$  (T. Ohtsuki, private communication) we calculated  $\lim_{L \rightarrow \infty} g(L) = 1.411$ , which is in very good agreement with our present result.

Assuming for square systems  $\sigma \equiv g/L^{(2-d)}$ , we have at the critical point  $g_c = \sigma_c$ . As mentioned above, for  $L > 20$  our critical conductance data do not exhibit any finite-size effect. We do not expect that the numerical analysis causes any inaccuracy of the estimation of  $g_c$ . Still, when comparing  $g_c$  with  $\sigma_c$ , we have to keep in mind that small corrections due to the properties of leads (we assume perfect leads without spin-orbit scattering) might be responsible for a small difference between  $\sigma_c$  and  $g_c$ . However, we do not expect this difference to be larger than  $\sim L^{-1}$  and neglect it for further purposes. Figure 5 shows the critical conductance distribution  $p(g)$ . We present data for disorder  $W = 5.84$  and system



**Figure 5.** Critical conductance distribution  $p(g)$ . Inset: the derivative  $\partial p(g)/\partial g$  which shows  $g_{\text{non-an}}$  to be slightly larger than 2.



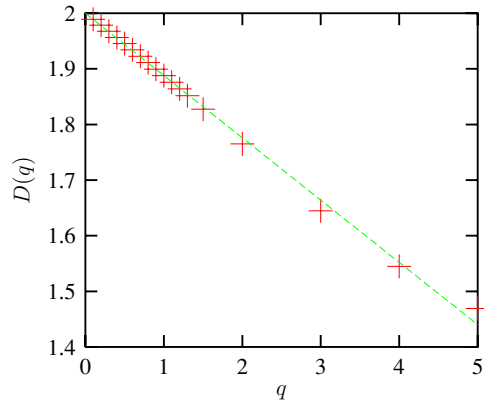
**Figure 6.** The  $f(\alpha(q))$ -distributions of ten critical eigenstates at  $E \approx 0$  and  $W_c = 5.83$  for  $q = 0.0, \pm 0.2, \pm 0.5, \pm 0.8, \pm 1.0, \pm 1.5, \pm 2.0$  and  $\pm 3.0$ . The parabolic fit is determined by  $\alpha_0 = 2.107 \pm 5 \times 10^{-3}$ . The inset shows the scaling of  $\ln P(1, \lambda)$  versus logarithm of the box size  $\lambda$ . Data are obtained from ten  $L = 260$  samples with different disorder realizations. The dashed line has steepness  $\tau(1) = 1.889$ .

size  $82 \leq L \leq 200$ . We see that indeed  $p(g)$  does not depend on the system size at the critical point. Besides the shape of the distribution, which is known also from previous studies, we want to point out the non-analytical behaviour of the distribution at the point  $g = g_{\text{non-an}}$  which is slightly larger than 2. This agrees with a qualitative estimation of Muttalib *et al* [35], although their work concerns quasi-one-dimensional systems with orthogonal symmetry.

### 3.3. Fractal dimensions and Fal'ko–Efetov relation

In figure 6 the  $f(\alpha(q))$ -distributions of ten critical eigenfunctions of systems with  $L = 260$  are shown together with a parabolic fit  $f(\alpha(q)) = d - (\alpha(q) - \alpha_0)^2/[4(\alpha_0 - d)]$  with  $\alpha_0 = \alpha(q = 0) = 2.107 \pm 0.005$ . This value is lesser than the one obtained previously for smaller sample size  $L = 150$  [9]. The inset exhibits the logarithm of  $P(1, \lambda)$  plotted versus





**Figure 7.** The generalized fractal dimension  $D(q)$  of a critical eigenfunction as a function of  $q$  calculated via the box probability (4). The linearity  $D(q) = 2 - q/k$  holds for  $q \lesssim 1.5$ , with  $k = 0.112$  leading to  $\sigma_c = 0.71e^2/h$ .

$\ln \lambda$  for  $2 \leq \ell \leq 25$ . The data belonging to critical eigenstates obtained from ten disorder realizations clearly obey power-law scaling.

From the multi-fractal analysis of critical eigenfunctions of  $L = 260$  samples, we obtained an information dimension  $D(1) = 1.889 \pm 0.001$  using ten different realizations with disorder strength  $W = 5.83$ . With these data, we checked the validity of a formula suggested by Fal'ko and Efetov [17] which should hold in the limit  $1 \leq \varrho Dh$ ,

$$D(q) = 2 - \frac{q}{\beta 4\pi^2 \varrho D \hbar}, \quad (13)$$

where  $\beta = 1/2, 1$  and  $2$  for orthogonal, unitary and symplectic symmetry, respectively. With  $\sigma = e^2 \varrho D$  (diffusion constant  $D$  and density of states  $\varrho$ ) we get

$$\sigma = \frac{q}{2\pi\beta[2 - D(q)]} \frac{e^2}{h}. \quad (14)$$

Taking  $q = 1$ , we found our results for  $\sigma_c$  and  $D(1)$  to be in very good agreement with (14). Therefore, this relation is fulfilled also for the symplectic symmetry in a similar manner as found previously for the quantum Hall case [36]. In comparing these results one has to keep in mind that in the present paper two spin channels were considered, i.e., the conductance has to be divided by 2, whereas in the QHE case only one spin direction was taken into account. Note also that the value of the disorder used in the multi-fractal analysis,  $W = 5.83$ , differs slightly from the critical disorder  $W_c = 5.838$ . Therefore, the conductance  $g(W = 5.83) \approx 1.43$  mentioned in [36] is a little larger than  $g_c$ . The validity of the proposed linear relationship (13) holds at least for  $q \leq 1.5$  which can be seen in figure 7 where  $D(q)$  calculated according to (4) is plotted versus  $q$ .

### 3.4. Comparison with Janssen's formula

A formula that connects fractal properties of critical eigenstates in square samples with the finite-size scaling variable  $\Lambda = \lambda_M/M$  of quasi-1D systems was conjectured by Janssen [18, 19]. At the critical point,  $\Lambda_c$  is scale independent and depends only on the Lipschitz-Hölder exponent  $\alpha_0$  and the spatial dimension of the system  $d = 2$

$$\Lambda_c = \frac{1}{\pi(\alpha_0 - d)}. \quad (15)$$

Equation (15) was reported to hold for 2D disordered systems in the quantum Hall regime [18, 37]. In the present symplectic system, however, it is not met by our results. Also, assuming the parabolic approximation to be valid, we can use the relation between  $D(1)$  and  $\alpha_0$ ,

$$D(1) = \alpha(1) = 4 - \alpha_0, \quad (16)$$

and connect (14) and (15) to obtain a relation between  $\Lambda_c$  and the critical conductivity

$$\sigma_c = \frac{\Lambda_c e^2}{2\beta h}. \quad (17)$$

Again, contrary to what is observed in the QHE regime, the relation (17) is not satisfied for a symplectic system described by the Ando model. A possible reason for this failure might be that in the present 2D model, the scaling parameter  $\Lambda_c$  may not represent the typical localization length of the system as has been assumed in [18].

#### 4. Conclusion

We studied the electrical two-terminal conductance and the spatial fluctuations of electron eigenfunctions near the metal–insulator transition of the two-dimensional Ando model. Using finite-size scaling, we obtained for this symplectic model a critical exponent  $\nu = 2.8 \pm 0.04$ , which governs the size dependence of the conductance in the critical regime, and a critical disorder  $W_c = 5.838 \pm 0.007$ . Our results for the critical conductance and for the fractal dimension of critical eigenstates confirm the validity of Fal’ko and Efetov’s prediction (13). However, comparison of the fractal dimensions with the critical value of the finite-size scaling parameter  $\Lambda_c$ , calculated for quasi-one-dimensional systems, indicates that Janssen’s formula (15) is not fulfilled by our data in the present model.

Unlike the localization length, the two-terminal conductance should be easily accessible in experiments. Our numerical confirmation of the relation between fractal dimensions of critical eigenfunctions and the critical conductance provides an additional argument for universality at the metal–insulator transition in 2D symplectic models.

#### Acknowledgments

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